Remarks on a Temporal Intuitionistic Fuzzy Logic

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In the memory of Acad. H. Hristov

Let \( p \) be a proposition and let \( V \) be a truth-value function, which juxtaposes to the proposition \( p \) and to the time-moment \( t \in T \) (\( T \) is a fixed set which we shall call “time-scale” and it is strictly oriented by the relation “\(<\)”) the ordered pair (c.f. [1]).

\[
V(p, t) = \langle \mu(p, t), \nu(p, t) \rangle
\]

Let

\[
T' = \{ t' \in T \text{ and } t' < t \}
\]

\[
T'' = \{ t'' \in T \text{ and } t'' > t \}
\]

We shall define for given \( p \) and \( t \) the operators

\[
P, F, H, G : [0, 1] \times [0, 1] \times T \to [0, 1] \times [0, 1] \times T,
\]

for which

\[
X(p, t) = X(\langle \mu(p, t), \nu(p, t) \rangle)
\]

for \( X \in \{ P, F, H, G \} \) and:

- \( V(P(p, t)) = \langle \mu(p, t'), \nu(p, t') \rangle \), where \( t' \in T' \) satisfies the conditions:
  
  (a) \( \mu(p, t') - \nu(p, t') = \max_{t* \in T} (\mu(p, t*) - \nu(p, t*)) \),

  (b) if there exist more than one such element of \( T' \), then \( t' \) is the maximal.

- \( V(F(p, t)) = \langle \mu(p, t''), \nu(p, t'') \rangle \), where \( t'' \in T'' \) satisfies the conditions:

  (a) \( \mu(p, t'') - \nu(p, t'') = \max_{t* \in T} (\mu(p, t*) - \nu(p, t*)) \),

  (b) if there exist more than one such element of \( T'' \), then \( t'' \) is the minimal.

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- $V(H(p, t)) = \langle \mu(p, t'), \nu(p, t') \rangle$, where $t' \in T'$ satisfies the conditions:
  (a) $\mu(p, t') - \nu(p, t') = \min_{t \in T'} (\mu(p, t) - \nu(p, t))$,
  (b) if there exist more than one such element of $T'$, then $t'$ is the maximal.

- $V(G(p, t)) = \langle \mu(p, t''), \nu(p, t'') \rangle$, where $t'' \in T''$ satisfies the conditions:
  (a) $\mu(p, t'') - \nu(p, t'') = \min_{t \in T''} (\mu(p, t) - \nu(p, t))$,
  (b) if there exist more than one such element of $T''$, then $t''$ is the minimal
  (see e.g. [2, 3]).

**Theorem 1:** For every proposition $p$ and for every time moment $t$:

(a) $V(H(p, t)) = V(N(P(N(p)), t)))$,

(b) $V(G(p, t)) = V(N(F(N(p)), t)))$.

**Proof:**

(a) $V(N(F(N(p)), t))) = \langle \mu(p, t'), \nu(p, t') \rangle$

where $t'$ is the maximal element of $T'$ for which:

$$\nu(p, t') - \mu(p, t') = \max_{t \in T'} (\nu(p, t) - \mu(p, t)).$$

Therefore, $t'$ is the maximal element of $T'$ for which:

$$\mu(p, t') - \nu(p, t') = \min_{t \in T'} (\mu(p, t) - \nu(p, t)),$$

i.e.,

$$\langle \mu(p, t'), \nu(p, t') \rangle = V(H(p, t)).$$

(b) is proved analogically. \qed

**Theorem 2:** For every two propositions $p$ and $q$, and for every time moment $t$:

(a) $H(p \supset q, t) \supset (P(p, t) \supset P(q, t))$,

(b) $G(p \supset q, t) \supset (F(p, t) \supset F(q, t))$,

(c) $N(P(N(p \supset q), t)) \supset (P(p, t) \supset P(q, t))$,

(d) $N(F(N(p \supset q), t)) \supset (F(p, t) \supset F(q, t))$.

are IFSs.

**Proof:**

(a) $V(H(p \supset q, t) \supset (P(p, t) \supset P(q, t))) =$

$$= H(\langle \max(\nu(p), \mu(q)), \min(\mu(p), \nu(q)), t) \rangle \supset \langle \mu(p, t_1), \nu(p, t_1) \rangle \supset \langle \mu(q, t_2), \nu(q, t_2) \rangle),$$

(\text{where $t_1$ and $t_2$ are both maximal elements of $T'$ for which the maximums of $\mu(p, t_1) - \nu(p, t_1)$ and of $\mu(p, t_2) - \nu(p, t_2)$ are achieved})
If contradiction.

Let for every $t'$ the maximal element of $T'$ for which the maximum of $\max(p, t'), \mu(q, t')$ is achieved.

$$
= \langle \max(v(p, t'), \mu(q, t')), \min(\mu(p, t'), \nu(q, t')) \rangle
$$

(\text{where } t' \text{ is the maximal element of } T' \text{ for which the maximum of } \max(v(p, t'), \mu(q, t')) - \min(\mu(p, t'), \nu(q, t')) \text{ is achieved})

$$
= \langle \max(v(p, t_1), \mu(q, t_2)), \min(\mu(p, t_1), \nu(q, t_2)) \rangle,
$$

Then we consider the expression

$$a = \max(v(p, t_1), \mu(q, t_2)), \min(\mu(p, t_1), \nu(q, t_2)) - \min(\mu(p, t_1), \nu(q, t_2), \max(v(p, t'), \mu(q, t'))).$$

If there exist $t_2 \in T'$ for which $\mu(p, t_2) \geq v(q, t_2)$:

$$a \geq \mu(p, t_2) - v(q, t_2) \geq 0.
$$

Let for every $t_2 \in T'$: $\mu(p, t_2) < v(q, t_2)$. Then, if there exist $t_1 \in T'$ for which $v(p, t_1) \geq \mu(q, t_1)$:

$$a \geq v(p, t_1) - \mu(q, t_1) \geq 0.
$$

Let for every $t_1 \in T'$: $v(p, t_1) < \mu(q, t_1)$. Then, if there exist $t' \in T'$ for which $\min(\mu(p, t'), v(q, t')) \geq \max(v(p, t'), \mu(q, t'))$:

$$a \geq \min(\mu(p, t'), v(q, t')) - \max(v(p, t'), \mu(q, t')) \geq 0.
$$

Let for every $t' \in T'$:

$$\min(\mu(p, t'), v(q, t')) < \max(v(p, t'), \mu(q, t'))$$

and let $t_0 \in T'$. If $\mu(p, t_0) \leq v(q, t_0)$, then $v(p, t_0) \leq v(q, t_0)$ and $v(q, t_0) < \mu(q, t_0)$, which is a contradiction.

If $v(p, t_0) < \mu(q, t_0)$, then $\mu(p, t_0) < v(q, t_0)$ and $\mu(q, t_0) < v(q, t_0)$, which is a contradiction. Therefore (a) is valid.

(b)-(d) are proved analogically.

From the above definition, it follows the validity of:

**Theorem 3:** For every two propositions $p$ and $q$, and for every time moment $t$:

(a) If $H(p, t)$ is an IFT, then $P(p, t)$ is an IFT;

(b) If $G(p, t)$ is an IFT, then $F(p, t)$ is an IFT;

From the equalities:

$$P(P(p, t), t) = P(p, t),$$

$$F(F(p, t), t) = F(p, t),$$

it follows the validity of:
Theorem 4: For every proposition $p$ and for every time moment $t$:

(a) $P(P(p, t), t) \supset P(p, t)$

(b) $H(p, t) \supset H(H(p, t), t)$

are IFTs.

Let $W' = \{t' / t' \in T' \& t' \leq t\}$, $W'' = \{t'' / t'' \in T'' \& t'' \geq t\}$. The operators $\overline{P}$, $\overline{H}$, $\overline{F}$ and $\overline{G}$ are defined as the respective above, but for $W'$ and $W''$ instead of $T'$ and $T''$. For them the above assertions are valid also.

Theorem 5: For every two propositions $p$ and $q$, and for every time moment $t$:

(a) If $\overline{H}(p, t)$ is an IFT, then $\overline{F}(p, t)$ and $(p, t)$ are IFTs,

(b) If $\overline{G}(p, t)$ is an IFT, then $\overline{F}(p, t)$ and $(p, t)$ are IFTs,

(c) If $\overline{H}(p, t) \& \overline{G}(p, t)$ is an IFT, then $\overline{P}(p, t) \lor (p, t) \lor \overline{F}(p, t)$ is an IFT,

(d) If $\overline{H}(p, t) \lor \overline{G}(p, t)$ is an IFT, then there exists $t^* \in T$ for which $(p, t^*)$ is an IFT,

where $(p, t)$ denotes the proposition $p$ at the time-moment $t$.

References

Original references as presented in Preprint IM-MFAIS-1-90
Facsimiles

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**Remark on a Temporal Intuitionistic Fuzzy Logic**

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Let $D$ be a proposition and let $V$ be a truth-value function, which juxtaposes to the proposition $p$ and to the time-moment $t \in T$ ($T$ is a fixed set which we shall call "time-space" and it is arbitrarily oriented by the relation $\preceq$) the ordered pair:  
\[(p(t), (v(p), (x, t)))\]

**Remark 1**. Let  
\[T' = \{ (v(t), (x, t)) \mid t \in T \} \]

we shall define for given $p$ and $t$ the operators $p(t)$, $p(t)$, $p(t)$, $p(t)$, $p(t)$, for example:  
\[p(t) = (p(t), (v(p), (x, t)))\]

for $x \in X$, $t \in T$, $0 \leq \alpha \leq 1$, and:

\[P(x, t) = (p(t), (v(p), (x, t)))\]

where $t \in T$ satisfies the conditions:  
\[(0, \alpha) \cup (0, 1) = (0, 1) \cup (0, 1) \]

**Remark 2**. If there exist more than one such element of $T'$, then $t'$ is the maximal:

\[P(x, t) = (p(t), (v(p), (x, t)))\]

where $t' \in T'$ satisfies the conditions:

\[(0, \alpha) \cup (0, 1) = (0, 1) \cup (0, 1) \]

**Remark 3**. If there exist more than one such element of $T'$, then $t'$ is the minimal:

\[P(x, t) = (p(t), (v(p), (x, t)))\]

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